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AN EFFICIENT METHOD FOR QUADRATICALLY CONSTRAINED QUADRATIC PROGRAMS WITH APPLICATION TO MULTI-OBJECTIVE RESPONSE SURFACE PROBLEMS

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Abstract

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A new efficient strategy, multi-response systems problem (MRP), is developed for solving Quadratically Constrained Quadratic Programs (QCQP) with an additional constraint in the form of a sphere or ball with radius ρ. This method can solve QCQP problems with m constraints and n variables. A unique feature of this method is that it can efficiently solve general quadratic problems without duality gaps.

The method applies a two-level dual approach to solve the problem. In the lower level, it solves a minimization problem where the objective function in the Lagrangian consisting of the quadratic constraints of the original QCQP and the constraint consists only of the spherical constraint. A trust-region based algorithm is used to solve the lower level problem and guarantees that the Hessian of the problem is positive definite.

In the upper level, an updating scheme is used to update the multipliers to improve the dual function in an attempt to eventually maximize it. The updating scheme used is equivalent to a projected *Newton's method* and combined with a projected *steepest ascent* method when necessary or when it is found advantageous. Since the dual function is concave. This updating scheme generally works well. Mechanisms are built-in

to ensure strict finite improvement of the dual function in each iteration, hence ensuring not only convergence to the dual solution but also a nice convergence rates.

Test results show the proposed algorithm work well for problems of various sizes, complexity and convexity, as long as they do not have duality gaps. Modifications of the algorithm are needed to handle non-convex problems with duality gaps.

The proposed algorithm is applied to generate efficient frontiers for MRP. To generate an efficient frontier for a multiple response surface problem (MRP), we solve a series QCQP constraint problems. Due to the nature of the algorithm where the duality gaps can not be handled, only certain parts of the efficient frontier can be generated by the method. There are parts where not all secondary objectives can be varied independently. To generate the part of the efficient frontier where all secondary objectives can be varied independently, modification of the algorithm have to be made in the same way that it would be modified to handle problems with duality gaps.