

Simultaneous Multi Curve Approximation with NURBS

by

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Abstract

The design of wing and turbine blades requires simultaneous approximation of sectional airfoils. These airfoils are specified as a collection of data points. From the perspective of flow performance the airfoils should be approximated to within a prespecified tolerance. Furthermore, to facilitate skinning, the curves must share a mutual degree and a mutual knot vector, i.e., they must be compatible.

This thesis aims to provide a method for creating a Non Uniform Rational B-spline (NURBS) skeleton of wing/blade from the sectional data to facilitate the skinning process and to serve as an aid in the design process. The goal of this method is to simultaneously fit sectional data with NURBS curves that approximate it to a pre-specified tolerance with a minimum number of parameters (knots, weights and control points).

This is achieved in two steps. First a skeleton of compatible curves that approximate the sectional data is constructed. Second, the curve approximation is modified to satisfy the tolerance requirement. The generation of the skeletal curves is based on nonlinear least square optimization. The method uses the BFGS descent direction to overcome the lethargic property and uses the condition number of the least square matrix to ensure the good behaviour of control points. The method for satisfying prespecified tolerance is based on identifying the knot span, that contains the most number of data points outside the pre-specified tolerance, and inserting additional knots in them. The process is continued until all points are within the prespecified tolerance. This method is called the tolerance based knot insertion method. This two step process is designed to ensure that the compatibility of the

skeletal curves is maintained at all times.

The method for fitting skeletal curves is tested on three single-section cases and three multi-section cases. The tests showed that, for the single-section cases, the maximum error decreased by a factor of a least five , and up to ten, from the initial maximum error. For multi-section cases, the maximum error decreased by a factor between two, and up to sixteen, from the initial maximum error. Based on these tests it is concluded that parametrization of data does not contribute to reduction in least square error. On the other hand, knots contribute the most to the reduction of least square error.

The tolerance-based knot insertion was tested on three skeletons and proved to be successful in satisfying the tolerance with relatively low number of parameters. The tests showed that proposed method reduced the numbers of control points of the compatible curves that satisfy the prespecified tolerances between 20% to 40% in comparison to the existing methods.